DETERMINATION OF NEWTON'S GRAVITATIONAL CONSTANT, G, WITH IMPROVED PRECISION

Status Report
covering the period
1 April 1966-30 September 1966
under NASA Grant NGR-47-005-022

Principal Investigators

J. W. Beams
Department of Physics

A. R. Kuhlthau
R. A. Lowry
H. M. Parker
J. P. Senter
Department of Aerospace Engineering and Engineering Physics

Research Laboratories for the Engineering Sciences

University of Virginia

Charlottesville

Report No. EP-4028-104-66U
December 1966

DETERMINATION OF NEWTON'S GRAVITATIONAL CONSTANT, G, WITH IMPROVED PRECISION

Status Report
covering the period
1 April 1966-30 September 1966
under NASA Grant NGR-47-005-022

Principal Investigators

J. W. Beams
Department of Physics
A. R. Kuhlthau
R. A. Lowry
H. M. Parker
J. P. Senter
Department of Aerospace Engineering and Engineering Physics

RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA

Report No. EP-4028-104-66U
December 1966

Copy No. 2

SECTION I
INTRODUCTION

This is the fourth semi-annual status report under this grant, the object of which is to apply new techniques to the laboratory determination of the Newtonian Gravitational Constant, G, with the expectation of improvement of the accuracy by about two orders of magnitude. The improvement is attributed to both a new approach to the measurement, which should eliminate or bring under better control most of the difficulties encountered in earlier methods, and to improvements in materials, metrology, fabrication procedures, etc., which should circumvent many problems and limitations previously encountered.

With reference to the schematic diagram of Figure 1, two small masses, m, are connected by a rigid horizontal rod. These are then placed in proximity to another pair of masses, M, and the gravitational interaction between the two sets of masses results in a pure torque acting on the m-system. A measure of this torque plus a knowledge of the separation distance, d, enables G to be determined. Previous experiments either measured the torque directly by observing the static deflection of a torsion fiber suspension or introduced variations on this method such as inducing resonant torsional oscillations in the m-system by suitably driving the large M system.

In the present proposed method the gravitational torque is maintained constant by detecting changes in the relative angular position β of the two mass systems and correcting the relative position by a servo mechanism to maintain β constant to the necessary accuracy. Thus the nearly constant torque being applied to the m-system results in a nearly constant angular acceleration of this system which can be permitted to act for an extended period of time. The ultimate measurement, then, is one of determining the angular speed of the m-system at the end of this extended period of time, i.e., of measuring large displacements and long times, both of which can be done accurately.

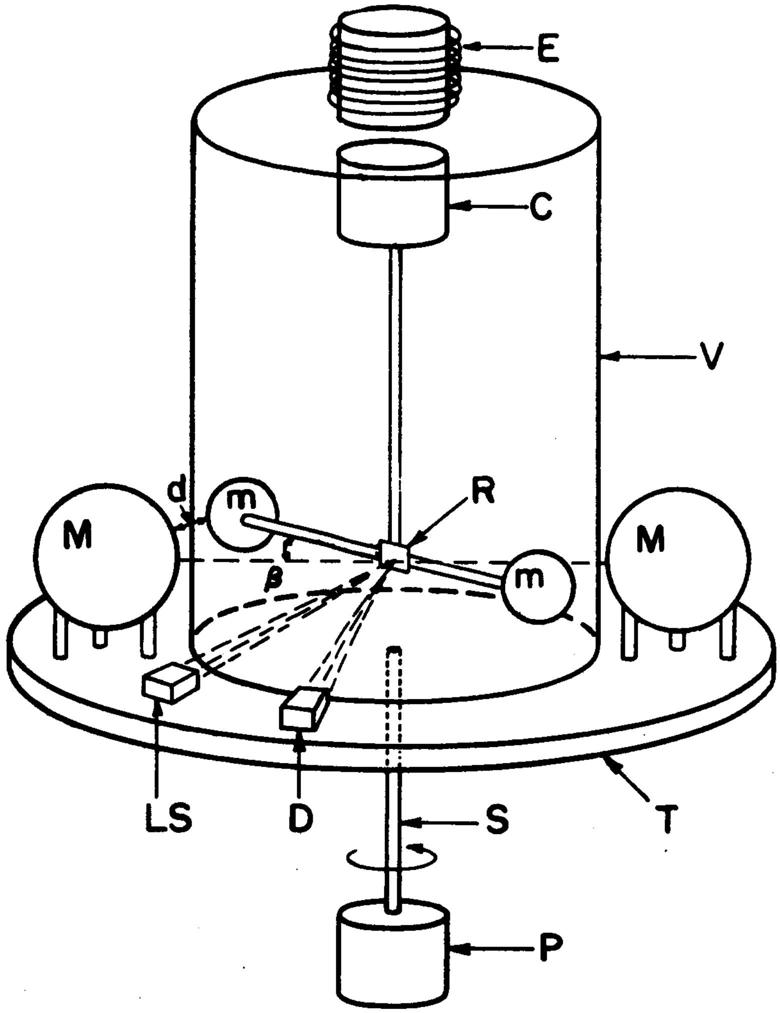

FIGURE I

SCHEMATIC OF APPARATUS

In designing the experiment, three methods for suspending the m-system have been considered. Two of these involve the use of magnetic suspensions and for future reference these will be designated Models A-1 and A-2. The other involves the use of a more conventional torsion fiber and this will be designated as Model B. Each system has many advantages and disadvantages in comparison with the others, and in fact it is at this stage impossible to make the final selection. They are all being studied more carefully as part of the initial phases of the program, and working models of each design are being constructed and evaluated.

For the orientation of the reader, the principal differences are:

Model A-1 and A-2--Magnetic Support

Again referring to Figure 1, the center of the rod connecting the small masses, m, is attached to a rigid vertical rod containing at its upper end, a small cylinder of magnetic material, C. This is freely suspended by the electromagnetic E [1]. The major advantage of the electromagnetic suspension models is that the precision to which the angle β must be held constant in order to get the desired precision in the determination of G is only 10^{-3} radians, an easy prospect. The major disadvantage is the rather elaborate magnetic shielding required.

In Model A-1 the coil of the electromagnet, E, is attached to the rotating table.

In Model A-2 the coil of the electromagnet is free of the rotating table and is fixed in the laboratory frame of reference.

The advantages and disadvantages between these two versions are more subtle. They are discussed in more detail in reference [2], and for the present purposes it need only be said that they do exist, are significant, and can only be evaluated by further experimentation.

Model B--Torsion Fiber

Model B replaces the electromagnetic suspension with a delicate torsion fiber mounted from the rotating table. As the m-system begins to rotate as a result of its interaction with the M system, the fiber support is rotated, due to the table rotation, and the balance always operates at essentially constant deflection (which will be as close as possible to the null point). The major disadvantage here is that this deflection angle and β, must be held constant to 10^{-6} radians in order to get accuracy in the G measurement comparable to what can be obtained by knowing β to 10^{-3} radians in the case of the magnetic support. The big advantage, of course, is the elimination of the magnetic field, and hence a minimization of magnetic effects.

At the present time the program is divided into three principle tasks, the status of each which is summarized in the following sections. These tasks are:

1. Theoretical Analysis
2. Apparatus Design--Details and Procurement of Components
3. Assembly of Apparatus and Development of Techniques.

SECTION II
CURRENT STATUS OF THE PROGRAM

A. Task 1--Theoretical Analysis

The theoretical calculations can be grouped into two categories.

1. Equation of Motion

A moderate amount of effort during this report period was directed to the general problem of data reduction, especially concerning the effects of laboratory fixed masses (and any other sources of $\sin 2\theta$ torques on the small mass system). As reported previously, the effect of a mass fixed in the laboratory gives rise to a term in the motion equation proportional to $\sin 2\theta$ provided that w, the ratio of the half-length of the small mass system to the distance to the fixed mass, is small. More exactly, the angular acceleration due to a laboratory mass can be expanded in an even power series in w and the coefficient of w^2 and higher powers additionally contain terms proportional to the sine of higher even multiples of θ. For the first order, the motion equation is

$$\ddot{\theta} = a + A \sin(2\theta + \phi) \tag{1}$$

where $\ddot{\theta}$ is the angular position of the small mass system, a is the constant acceleration due to the interaction of the two mass systems, and A and ϕ in the last term can be found in terms of the size and position of the laboratory fixed mass. It is straightforward to show that a collection of discrete laboratory fixed masses (and hence a continuous distribution) may be replaced by a single effective mass at an appropriate position, i.e.,

$$\sum_j A_j \sin(2\theta + \phi_j) = A_{eff} \sin(2\theta + \phi_{eff})$$

where

$$A_{eff} = \left[(\sum_j A_j \cos \phi_j)^2 + (\sum_j A_j \sin \phi_j)^2 \right]^{1/2}$$

$$\phi_{eff} = \tan^{-1} \left[(\sum_j A_j \sin \phi_j) / (\sum_j A_j \cos \phi_j) \right] .$$

A solution to equation (1) considered suitable for data reduction has been obtained ($t(\theta)$ via the "energy" integral). To first order in A, one has

$$t_n = \alpha \left\{ \sqrt{1 + 4\pi y n} - 1 \right\} + 2\pi A y^3 \alpha^3 \left\{ (S_1 \cos\phi + S_2 \sin\phi)\, n + (S_3 \cos\phi + S_4 \sin\phi) n^2 + \ldots \right\} \tag{2}$$

where

n = the number of revolutions ($\theta_n = 2\pi n$) of the small mass system

t_n = the time for n revolutions

$\alpha = \omega_0/a$

$y = a/\omega_0^2$.

The odd subscripted S are infinite series in even powers of y (order 1 for $y \ll 1$).

The even subscripted S are infite series in odd powers of y (order y for $y \ll 1$).

It is straightforward if tedious to derive the higher order terms of equation (2). Obviously the retention of the first curly bracket term on the right corresponds to the constant acceleration case and the second curly bracket term gives the first order effect of a laboratory fixed mass distribution.

Equation (2) was the basis for the initial consideration of the data reduction problem. One could anticipate a least squares fitting of data to equation (2), and consequent estimates of probable errors in the values of the fitted parameters (the desired quantity, G, is buried in $a^{-1} = \alpha^2 y$), the assumption being that if the laboratory fixed mass effect is significant the procedure enables it to be taken into account. Thus a computer program was developed to do a (nonlinear) least squares fit to equation (2). Two sorts of test procedures were initiated.

First, theoretical (or numerical) tests were made in the following general fashion. Values of the parameters α, y, A, and ϕ may be specified and inserted into equation (2) to generate "perfect" data sets (t_n, n) and then fed to the least squares program to produce "best" values of the parameters. Alternately, some noise may be added to the perfect data and the method used was a simple truncation of the t_n (n always will have integer values). A nonlinear least squares algorithm is an iterative procedure and starting values for the parameters are inputs to the program. These theoretical tests were less than satisfactory. For some cases, convergence to reasonable results occurred rapidly; in other cases convergence was very slow, and in some cases convergence (apparently) never occurred. Often exclusive dependence on starting values was exhibited. The reasons for this numerical pathology are not understood. The general conclusions are that this program, which was a Burroughs library nonlinear least squares procedure, is not adequate for equation (2) and that the reason is probably associated with the fact that in equation (2) t_n has a weak dependence on A and ϕ as compared with α and y.

Secondly, when the apparatus just became operational (with the nonprecision mass systems) experiments were designed to test the laboratory fixed mass effect. Runs were made with and without a large (≈ 100 lbs.) laboratory mass fairly close (≈ 50 cm) to the apparatus, and data for three such pairs of runs were selected. It was recognized that the data might be excessively noisy for several reasons; e.g., the small mass system was supported by a metal (tungsten) fiber whose torsion constant could be noisy and the friction in the table bearing was larger than anticipated and excessively loaded the drive motor. Nevertheless, an attempt was made to fit the data to equation (2). One set of data converged rapidly. One set converged after many iterations. One set did not converge after several thousand iterations. The two sets of data (each set consisting of data with and without the external mass) which converged gave results which were neither internally consistent nor physically reasonable, especially with respect to the fitted values of A. In view of the difficulties experienced with the "perfect" data tests, it could not be concluded whether it was simply a matter of noise in the experimental data or whether the least squares program is pathological, or both.

One of the auxilary objectives of the effort described above was and is to arrive at reasonable estimate of the laboratory fixed mass effect to be expected in the final precision experiments. Attempts to estimate the effect from fundamental theoretical considerations proved frustrating initially for the following reason. The contribution to the torque on the small mass system of a constant density mass distribution in an element of solid angle $d\Omega$ between a minimum distance r_0 and a maximum distance r_∞ diverges logarithmically as the distance r_∞ goes to infinity. On the other hand, the collection of effects from elements of solid angles at different angles must be large since for an axially (or spherically) symetric mass distribution the total effect vanishes. Finally, however, a physically sound model was designed to yield a upper limit on A. If one assumes that exterior to the apparatus the entire space (fixed in laboratory) is either empty or occupied by mass of density, ρ, then it seems physically sound to assume that the maximum effect results when the entire mass is distributed so that mass centers of two spherical distributions lie symmetrically on opposite sides of the center of the small mass system. Thus, one assumes that the worst possible case corresponds to the entire external mass being two constant density spheres on opposite sides of the apparatus. Then letting the sphere size and mass go to infinity gives a finite effect corresponding to

$$A_{max} = 4\pi\rho G$$

or taking $\rho = 5\,gm/cm^3$

$$A_{max} \approx 4 \times 10^{-6}\ rad/sec^2.$$

Realistically, one may expect A to be significantly smaller than this.

Current effort is proceeding in two directions. First, a systematic investigation is underway of the errors involved in the fitted values of a and ω_0, as a function of a, ω_0, A, ϕ, and n, by computing perfect data from equation (2) and fitting to equation (2) with the laboratory mass effect term deleted. Since a good upper bound on A has been found, this seems to be a reasonable procedure. It is possible, though currently considered unlikely, that the laboratory fixed mass effect may be neglected entirely, i.e., Δt_n (n), or time for one revolution or a fixed number of revolutions, is to be used for least squares fitting. Such a change may favorably change the numerical character of the problem (as well as correspond more closely to the raw experimental data) though, obviously, it does not change any essential weakness of the dependence on A.

Secondly, as the apparatus gets back into operation additional laboratory mass effect experiments will be run to (hopefully) give less noisy data and determine the actual A with some precision.

2. Analysis of the Gravitational Torque on a Cylinder

A cylindrical small mass system suspended about an axis perpendicular to its own axis has several advantages over the dumbell type small mass system. (1) It is easier to fabricate, (2) it is more rigid and durable, (3) by using an optical material, density variations can be detected and measured, and (4) by making the ends of the cylinder optically flat and reflecting, interference techniques can be used for centering.

The disadvantage of the cylindrical small mass system is that the mathematical analysis of the gravitational torque produced is much more involved than in the case of the dumbell small mass system.

Fortunately, the analysis of this problem can be separated into two distinct parts. In the first part the gravitational potential of a cylinder is calculated. In the second part, this potential is used to determine the torque and acceleration the cylinder will experience.

a. The Gravitational Potential of a Cylinder

Referring to Figure 2, the gravitational potential on the z axis for $z > C$ of a ring of mass m, radius r and located at $z = C\cos\alpha$ is

$$\phi_{ring}(R, O) = \frac{mG}{(R^2 + C^2 - 2RC\cos\alpha)^{1/2}} \quad . \tag{3}$$

This can be expanded in Legendre polynomials to give

$$\phi_{ring}(R, O) = mG \sum_{\ell=0}^{\infty} \frac{C^\ell}{R^{\ell+1}} P_\ell(\cos\alpha) \quad . \tag{4}$$

The gravitational potential at any point in space is obtained by multiplying each member of this series by $P_\ell(\cos\theta)$ [6].

$$\phi_{ring}(R, \theta) = mG \sum_{\ell=0}^{\infty} \frac{C^\ell}{R^{\ell+1}} P_\ell(\cos\alpha) P_\ell(\cos\theta) . \tag{5}$$

The potential ϕ for a cylinder is obtained by writing this in differential form and integrating over r and z.

$$\phi(R, \theta) = \frac{2\pi\rho G}{R} \sum_{\ell=0}^{\infty} \frac{P_\ell(\cos\theta)}{R^\ell} \int_{-L/2}^{+L/2} \int_0^a C^\ell P_\ell(\cos\alpha) r\, dr\, dz \tag{6}$$

where ρ is the density of the cylinder.

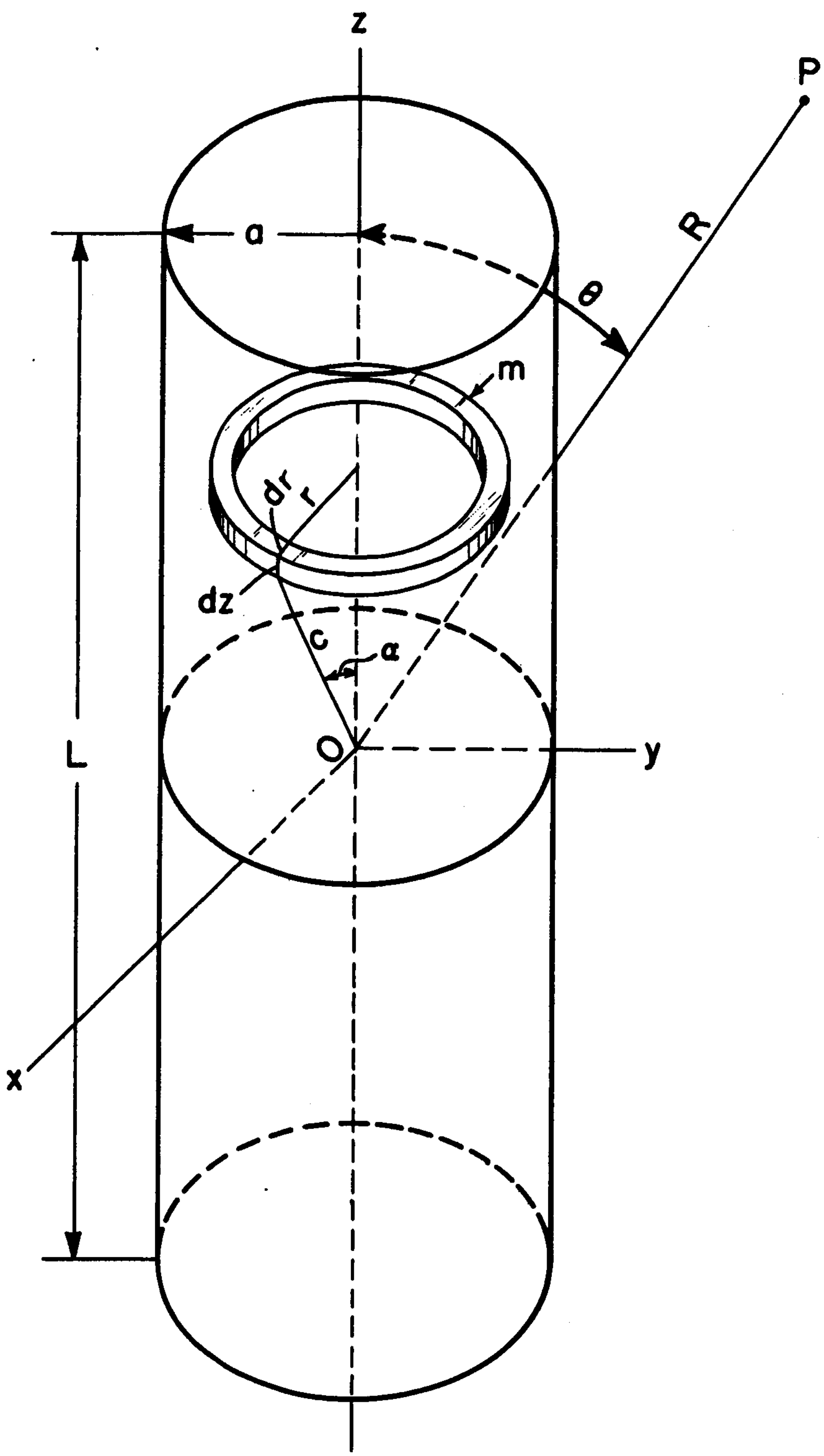

FIGURE 2

GEOMETRY FOR DISCUSSION OF GRAVITATIONAL TORQUE ON A CYLINDER

First perform the integration over r. The part of equation (6) which is involved in this integration is

$$R = \int_0^a C^{\ell} P_{\ell}(\cos\alpha)\, r\, dr \ . \tag{7}$$

This is most conveniently integrated by using Rodrigues' formula for $P_{\ell}(\cos\alpha)$ and expanding it by means of the binomial expansion. Let $\mu = \cos\alpha$, then

$$P_{\ell}(\mu) = \frac{1}{2^{\ell}\ell!} \frac{d^{\ell}}{d\mu^{\ell}} (\mu^2 - 1)^{\ell} \tag{8}$$

$$P_{\ell}(\mu) = \sum_{k=0}^{\ell} \frac{(-1)^k}{2^{\ell}\ell!} \binom{\ell}{k} \frac{d^{\ell}}{d\mu^{\ell}} \mu^{2(\ell-k)} \tag{9}$$

where

$$\binom{\ell}{k} = \frac{\ell!}{k!(\ell-k)!} \ . \tag{10}$$

In order to take this derivative, observe that

$$\frac{d^p X^n}{d^p X} = \frac{n!}{(n-p)!} X^{n-p} \ . \tag{11}$$

This yields

$$P_{\ell}(\mu) = \sum_{k=0}^{\ell} A_{k\ell}\, \mu^{\ell-2k} \tag{12}$$

where

$$A_{k\ell} = \frac{(-1)^k}{2^{\ell}\ell!} \binom{\ell}{k} \frac{(2\ell-2k)!}{(\ell-2k)!} \ . \tag{13}$$

From the geometry of Figure 2, it can be seen that

$$\mu = \cos\alpha = z/C \tag{14}$$

and

$$C^2 = z^2 + r^2 . \tag{15}$$

Substitution of equations (12), (13), (14), and (15) into equation (7) gives

$$R = \sum_{k=0}^{\ell} A_{k\ell} \, z^{\ell - 2k} \int_0^a (z^2 + r^2)^k \, r \, dr \, . \tag{16}$$

Straightforward integration yields

$$R = \sum_{k=0}^{\ell} \frac{A_{k\ell}}{2(k+1)} z^{\ell - 2k} \left[(z^2 + a^2)^{k+1} - z^{2(k+1)} \right] . \tag{17}$$

Substitution of equation (17) into equation (6) gives for the gravitational potential

$$\phi(R, \theta) = \frac{\pi \rho G}{R} \sum_{\ell=0}^{\infty} \sum_{k=0}^{\ell} \frac{A_{k\ell}}{(k+1)} \frac{P_\ell(\cos\theta)}{R^\ell} \left[\int_{-\frac{L}{2}}^{+\frac{L}{2}} (z^2+a^2)^{k+1} z^{\ell - 2k} dz - \int_{-\frac{L}{2}}^{+\frac{L}{2}} z^{\ell+2} dz \right] . \tag{18}$$

The integration over z must now be performed. The terms in the square bracket are the ones of concern. Applying the binomial expansion to the first term gives

$$Z = \sum_{j=0}^{k+1} \binom{k+1}{j} a^{2j} \int_{-\frac{L}{2}}^{+\frac{L}{2}} z^{\ell - 2j+2} dz - \int_{-\frac{L}{2}}^{+\frac{L}{2}} z^{\ell+2} dz \, . \tag{19}$$

By inspection it can be seen that the $j=0$ term of Z is equal to zero, so it can be rewritten as

$$Z = \sum_{j=1}^{k+1} \binom{k+1}{j} a^{2j} \int_{-\frac{L}{2}}^{+\frac{L}{2}} z^{\ell-2j+2}\, dz . \tag{20}$$

This integrates to

$$Z = \begin{cases} 2 \sum_{j=1}^{k+1} \binom{k+1}{j} \frac{1}{(\ell-2j+3)} \left(\frac{2a}{L}\right)^{2j} \left(\frac{L}{2}\right)^{\ell+3} & \text{when } \ell \text{ is even} \\ 0 & \text{when } \ell \text{ is odd} \end{cases} \tag{21}$$

The expression for the potential now becomes

$$\phi(R, \theta) = \frac{2\pi\rho G}{R} \sum_{\ell=0}^{\infty} \sum_{k=0}^{\ell} \sum_{j=1}^{k+1} \frac{A_{k\ell}}{(k+1)(\ell-2j+3)} \binom{k+1}{j} \left(\frac{2a}{L}\right)^{2j} \left(\frac{L}{2R}\right)^{\ell} \left(\frac{L}{2}\right)^{3} P_{\ell}(\cos\theta) \tag{22}$$

where ℓ can only take on even values.

In terms of the mass of the cylinder

$$m = \rho\pi a^2 L \tag{23}$$

the gravitational potential is

$$\phi(R, \theta) = \frac{mG}{R} \sum_{\ell=0}^{\infty} \sum_{k=0}^{\ell} \sum_{j=1}^{k+1} A_{jk\ell} \left(\frac{2a}{L}\right)^{2(j-1)} \left(\frac{L}{2R}\right)^{\ell} P_{\ell}(\cos\theta) \tag{24}$$

where

$$A_{jk\ell} = \frac{A_{k\ell}}{(k+1)(\ell-2j+3)} \binom{k+1}{j} . \tag{25}$$

b. The Angular Acceleration of a Cylinder

The force on a mass, M, located at the point P can be derived from the expression for the potential by taking the gradient and multiplying by M.

$$F = M \nabla \phi \, . \tag{26}$$

The component of force perpendicular to $\vec{R}$ is the only one which gives rise to a torque about O. In spherical coordinates, this component is given by

$$F_\theta = \frac{M}{R} \frac{\partial \phi}{\partial \theta} \hat{\theta} \, . \tag{27}$$

When two masses are placed diametrically opposite each other at (R, θ) and $(R, \theta + \pi)$ a pure torque about the x-axis results given by

$$T = 2M \frac{\partial \phi}{\partial \theta} \, . \tag{28}$$

Applying this to equation (24) gives

$$T = \frac{2mMG}{R} \sum_{\ell=0}^{\infty} \sum_{k=0}^{\ell} \sum_{j=1}^{k+1} A_{jk\ell} \left(\frac{2a}{L}\right)^{2(j-1)} \left(\frac{L}{2R}\right)^{\ell} \frac{\partial}{\partial \theta} [P_\ell(\cos \theta)] \, . \tag{29}$$

The angular acceleration is given by

$$\dot{\omega} = \frac{T}{I} \tag{30}$$

where I is the moment of inertia. For a cylinder,

$$I = \frac{1}{3} m \left(\frac{L}{2}\right)^2 \left[1 + \frac{3}{4} \left(\frac{2a}{L}\right)^2\right] . \tag{31}$$

So, the angular acceleration of a cylinder will be

$$\dot{\omega} = 24 \frac{MG}{RL^2} [1 + 3(\frac{a}{L})^2]^{-1} \sum_{\ell=0}^{\infty} \sum_{k=0}^{\ell} \sum_{j=1}^{k+1} A_{jk\ell} (\frac{2a}{L})^{2(j-1)} (\frac{L}{2R})^{\ell} \frac{\partial}{\partial\theta}[P_\ell(\cos\theta)]. \tag{32}$$

This is the expression that will be used to determine G.

Most of the conclusions reached concerning a dumbell small mass system are valid for a cylindrical small mass system.

1. For a given M and R, $\dot{\omega}$ is zero at $\theta=0$ and $\theta=\pi/2$* and has a maximum at some intermediate angle. The monopole term ($\ell=0$) naturaly is zero. The $\ell=4$ term is approximately one hundredth of the $\ell=2$ term. Thus most of the properties of equation (32) can be ascertained by studying the term for which $\ell=2$. This term is

$$\dot{\omega}_2 = 72 \frac{MG}{RL^2} [1 + 3(\frac{a}{L})^2]^{-1} \sum_{k=0}^{2} \sum_{j=1}^{k+1} A_{jk2} (\frac{2a}{L})^{2(j-1)} (\frac{L}{2R})^2 \sin\theta\cos\theta \tag{33}$$

which has a maximum at $\theta = 45°$.

2. The mass of the small m-system is unimportant since the angular acceleration is completely independent of it.

B. Task II - Apparatus Design and Procurement of Components

The following information supplements that already given in previous reports [3], [4], [5] concerning the various subsystems of the apparatus.

1. Large Mass System

As indicated in the previous report [5], an order for the fabrication of the tungsten spheres has been placed with the Y-12 plant

* Note that the angle θ as used here is the same as the angle β as shown in Figure 1, and used in previous analyses.

of USAEC, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee. This order has not been fulfilled due to difficulties encountered by the fabricator in obtaining the proper tungsten material from their supplier. No definite date for the delivery of the spheres has been set. In the meantime, however, the fabricator has supplied two precision brass spheres of the required diameter which will suffice for the preliminary work with the prototype apparatus. Since the precision tungsten spheres actually will not be needed until the final precision measurements are made, this delay in their fabrication should cause little or no delay in the final measurements.

2. Small Mass System

The decision has been made to use a quartz rod of a nominal 1.5 inch length and 0.25 inch diameter suspended about an axis perpendicular to its own axis and inquiries concerning the possibility of using a laser crystal have been directly to several optical companies. A prototype small mass system has been made from aluminum and will soon be incorporated into the prototype apparatus No. 2.

The final decision on the dimensions of the small mass system was delayed until an expression for the acceleration of a cylinder due to gravitational forces was available, with the idea of minimizing the effect of masses fixed in the laboratory on the equation of motion by choosing the proper length to diameter ratio for the cylinder. However, when the maximum value of this effect was found to be so small, this ratio was no longer considered critical and other factors such as rigidity, ease of fabrication, and performance determined the final dimensions.

3. Rotary Table

The precision spindle upon which the rotary table is mounted has been an item of considerable concern during a sizable portion of the period covered by this report. Measurements were made of the friction in the spindle and it was observed that it had increased considerably since installation. In fact at one point in a revolution there was a discontinuous jumps in the friction which resulted in the servo drive system not being able to track properly due to inadequate frequency response. The spindle was repaired, under warranty, by the supplier with the explanation that the spindle was not properly lubricated. The rotary table is now back in operation and seems to be performing properly.

4. Servo Drive System

During the development of the servomechanism, it was situated in the clean room in close proximity to the rotary table so that the performance of the rotary table could be observed during its optimization. However, since it is desirable to minimize the effects of the mass and electromagnetic fields associated with the servo drive system, once it was working properly, it was moved approximately 15 feet from the rotary table so that is is now outside the clean room. The electrical leads to the rotary table are inside shielded conduits. The servomechanism is preforming satisfactorily at this increased distance.

5. Measurement of Period of Rotation of Rotary Table

The method chosen to make this measurement was described in detail in the last report [5]. As indicated, the theoretical resolution of this system is just equal to the desired resolution, which means that refinements are necessary before the system can be considered dependable. One consideration which could result in the theoretical resolution not being attained is the fact that the rotary table is connected to the earth while the sensor is connected to the building, resulting in a decoupling of the two systems. A more satisfactory approach would be one in which all of the apparatus necessary for measuring the period of rotation of the rotary table is placed on the surface plate thus coupling the measuring system with the rotary table. If and when the present apparatus proves unsatisfactory, such a system will be incorporated into the experiment.

6. Suspension - Model B

This is the torsion fiber suspension. A prototype model was in operation at the time of the last report. Since then this model has been used in experiments aimed at detecting the effect of a mass fixed in the laboratory on the equation of motion. The results of these experiments are discussed in Section A-1.

7. Suspension - Model A

This is the model employing the free magnetic suspension. The suspension system is being throughly bench tested before insertion into prototype apparatus No. 2 described in the previous report.

The servomechanism itself has been operational for some time, but difficulties were encountered in obtaining spheres which would rotate freely. Previous reports that magnetic supports are frictionless have been concerned with the case where the sphere was rotating through a complete revolution and angular velocity measurements were made at a particular point in a revolution. This corresponds to magnetic support model A-2. The fact that angular velocity measured in this manner is constant simply indicates that the magnetic field is conservative. However, it has been found that angular velocity may vary considerably during one revolution. In fact, when the initial angular velocity is slow enough, the sphere may execute torsional oscillations. This indicates that when the sphere is fixed relative to the solonoid as in support model A-1, there will be a torque on the small mass system due to the magnetic support. This torque can be minimized by supporting the sphere close to its null point of oscillation and by selecting a sphere whose period of oscillation is large. Spheres are now on hand whose period approaches 20 minutes. When displaced from the null point by 10^{-3} radians these spheres produce a torque which is 10^{-2} of the torque expected from gravitational interactions. Further reduction of this torque is impractical, so the magnetic suspension will be inserted into prototype apparatus No. 2 in the near future. In the measurements of the torque on the small mass system, a tare method will be used to eliminate the torque due to the magnetic support. The acceleration of the rotary table will be measured with and without the presence of the large masses and the difference in acceleration determined. This will correspond to the acceleration produced by the large masses alone.

C. Task III - Assembly of Apparatus and Development of Techniques

1. Prototype Apparatus No. 1

This uses a Model B or torsion fiber support and has been in operation for some time. It has been operated extensively for the purpose of refining the servo drive system, evaluating the friction in the spindle, and attempting to detect the effect of masses fixed in the laboratory. Most of the experiments performed to date have utilized a tungsten fiber suspension which has a high noise level. This has just been replaced by a 25 micron quartz fiber 30 cm long which should have a substantially lower noise level. Experiments are under way now to determine the suitability of this fiber.

2. Prototype Apparatus No. 2

This prototype model is still under construction. It was designed specifically for a magnetic suspension of the small mass system (Model A) and a detailed description was included in the last report [5]. The A-frame, slip ring assembly, and ion pump have been installed and checked out. The magnetic support system will be installed as soon as it has been thoroughly bench tested.

A decision has been made to place the tungsten spheres on a quartz plate so as to minimize thermal effects. An order for the quartz plate was placed on October 18, 1966. As soon as it is received, it will be incorporated into the apparatus.

3. Metrology

The problem of measuring the distances involved to the required precision is well in hand although no actual experience has been obtained. It has been decided that white light interferometers can be utilized for all the necessary measurements. These have the advantage that a unique black fringes appears in the interference pattern which can be used as a reference when transferring measurements from one piece to another. Thus it will be possible to compare the distance under consideration to a very accurate gauge block. Referring to Figure 3, two interferometers mounted on a U-frame are used to obtain interference fringes off both ends of a gauge block. The black fringe of both interferometers is placed in the center of the circular interference pattern and the rotary table is turned through 90°, so that fringes are obtained off the outer surface of the spheres, and the new positions of the black fringes recorded. The shift in position of the black fringes in dependent on the difference in length of the gauge block and the distance between the outer surface of the spheres and can be measured in microinches.

A single white light interferometer can also be used to check how accurately the small mass system rotates about its own axis. The interferometer is fixed on the turntable and fringes gotten off one end of the quartz rod. Then the rod is rotated until fringes are obtained off the other end. The difference in these two sets of fringes can be used to determine how far the axis of rotation is from the center of the rod.

In a similar manner, a single interferometer can be used to align the axis of rotation of the spindle and the small mass system. In this case, the interferometer is fixed on the rotary table and fringes are obtained off one end of the quartz rod. Then the rotary table is turned through 180° until fringes are seen off the other end of the rod. The difference in these fringe systems enables one to calculate the misalignment of the two axes.

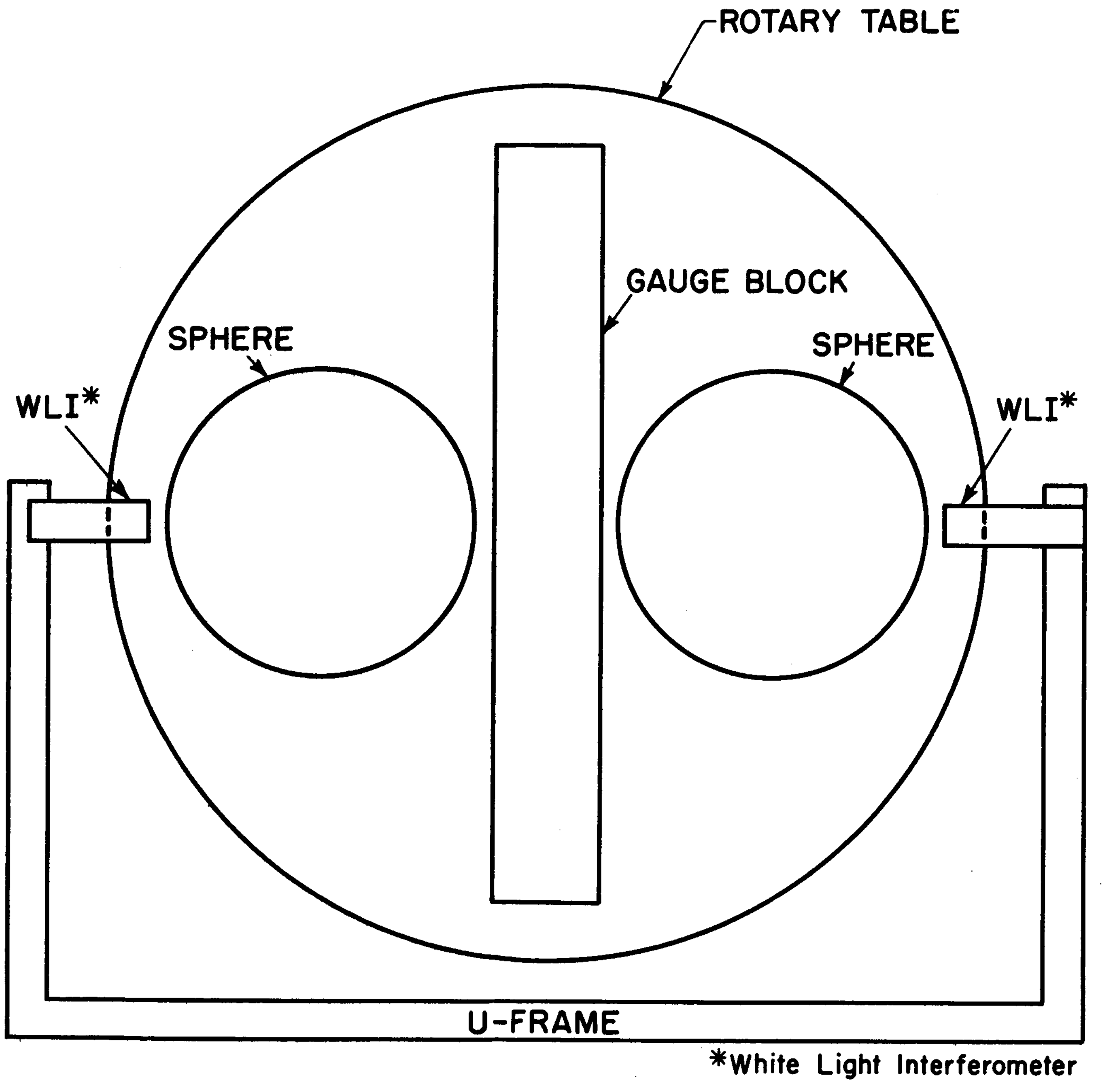

FIGURE 3

SETUP FOR MEASURING DISTANCE BETWEEN SPHERES

SECTION III
STAFF

The principal investigators gratefully acknowledge the assistance of the following staff members.

Dr. J. P. Senter, a senior physicist on the research staff of the Department of Aerospace Engineering and Engineering Physics who serves full time as the project manager of this program and who has been responsible for the refinement of the various techniques and their integration into an operational system.

Drs. John Boring and Robert Humphris, also members of the senior research staff of the Department of Aerospace Engineering and Engineering Physics, who have assisted in the conceptual design of many phases of the experiment in its formulative period.

Mr. James Dickerson, Design Engineer, who has developed the detailed specifications for many of the components procured commercially, and who has been responsible for the layout, construction and assembly of the two prototypes.

Messers. William Towler and Gerald Fisher, Electronic Engineers, who have developed the servo control for the rotary table, the magnetic suspension system, and the technique for measuring the period of rotation of the table.

Mrs. Jo Anne Kramer, a graduate student in Aerospace Engineering, who is performing the detailed calculations on many of the problems.

Finally much gratitude is due to Drs. A. V. McNish, T. R. Young and members of their staffs at the National Bureau of Standards, and to Messers. Roger Hibbs and Ed Bailey at the Y-12 Plant, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee, for their time and interest in the project which has resulted in many helpful suggestions and deeds.

SECTION IV
EXPENDITURE

		April 1, 1966-September 30, 1966	Total
1.	Salaries and Labor	$13,181.41	$47,657.71
2.	Employee Benefits	1,395.03	4,350.18
3.	Machine Shop Services	1,368.90	7,748.77
4.	Reports Group Services	131.94	553.23
5.	Travel	242.20	613.72
6.	Materials and Supplies	827.23	5,609.12
7.	Communications	403.56	547.92
8.	Equipment	3,044.67	21,915.21
9.	Encumbrances	7,621.00	7,621.00
	SUBTOTAL	$28,215.94	$97,616.86
10.	Indirect Costs	5,643.18	19,523.37
	TOTAL EXPENDITURES	$33,859.12	$117,140.23

REFERENCES

1. See e. g.,

   Beams, J. W., "Ultra High Speed Rotation," Sci. American, 204, 134, April 1961.

   Beams, J. W., "High Speed Rotation," Physics Today, 12, 20, July 1959.

2. Beams, J. W., Kuhlthau, A. R., Lowry, R. A. and Parker, H. M., "Determination of Newton's Gravitational Constant, G, with Improved Precision," University of Virginia, Research Laboratories for the Engineering Sciences, Proposal No. EP-NASA-125-64U, submitted to NASA, June 1964.

3. Beams, J. W., Kuhlthau, A. R., Lowry, R. A., and Parker, H. M., "Determination of Newton's Gravitational Constant, G, with Improved Precision," Status Report No. 1, June 1965; University of Virginia, Research Laboratories for the Engineering Sciences, Report EP-4028-101-65U, NASA-CR-63796.

4. Beams, J. W., Kuhlthau, A. R., Lowry, R. A., and Parker, H. M., "Determination of Newton's Gravitational Constant, G, with Improved Precision," Status Report No. 2, December 1965; University of Virginia, Research Laboratories for the Engineering Sciences, Report EP-4028-102-65U.

5. Beams, J. W., Kuhlthau, A. R., Lowry, R. A., and Parker, H. M., "Determination of Newton's Gravitational Constant, G, with Improved Precision," Status Report No. 3, May 1966; University of Virginia, Research Laboratories for the Engineering Sciences, Report EP-4028-103-66U.

6. Jackson, J. D., "Classical Electrodynamics," John Wiley and Sons, p. 64 (1962).

DISTRIBUTION LIST

Copy No.	
1 - 6	National Aeronautics and Space Administration Technical Reports Office Office of Grants and Research Contracts Washington, D. C. 20546
7	Dr. William Brunk Office of Grants and Research Contracts NASA Washington, D. C. 20546
8 - 9	J. W. Beams
10	A. R. Kuhlthau
11	R. A. Lowry
12	H. M. Parker
13	J. P. Senter
14	J. W. Boring
15	R. R. Humphris
16	L. R. Quarles
17 - 23	RLES Files
24 - 25	University Library, Attn: J. C. Wyllie